PEDESTAL — H-mode Pedestal Module

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This file documents a module called `PEDESTAL`, which can be used to predict the pedestal of type I ELMy H-mode plasma. The `PEDESTAL` module implements the models for the L-H transition, pedestal density and pedestal temperature. Note, the `PEDESTAL` module has been tested and used when compiled with flags such that double precision is used.

1 L-H transition model

1.1 Power threshold

In the empirical model developed by Y. Shimomura [1], it is assumed that plasma makes a transition from L-mode to H-mode when the power crossing the separatrix ($P_{\rm loss}$) rises above a power threshold ($P_{\rm L-H}$):

$$P_{\rm loss} > P_{\rm L-H}. \tag{1}$$

The power threshold for the transition to H-mode is calculated using the following formula,

$$P_{\rm L-H} = 2.84 A_H^{-1} B_{\rm T}^{0.82} R^{1.0} \bar{n}_{e20}^{0.58} a^{0.81} [MW] \tag{2}$$

where A_H is the hydrogenic mass in AMU, $B_{\rm T}$ is the vacuum toroidal magnetic field at major radius R along the flux surface in Tesla, R is the major radius in meters, $\bar{n}_{e20}$ is the line average density in units of 10^{20} $\rm m^{-3}$ and a is the minor radius in meters. The RMS deviation of this model is 27%.

2 H-mode density model

2.1 A simple empirical model

In this empirical model [2, 3], it is assumed that the pedestal density is proportional to the line averaged electron density. It was found that the best fit to data from the ITPA pedestal database version 3.1 is

$$n_{\rm ped} = 0.71 \bar{n}_e, \tag{3}$$

where $n_{\rm ped}$ is the pedestal density in the unit of $\rm m^3$ and $\bar{n}_e$ is the line averaged electron density in the unit of $\rm m^3$. When this model is compared with the 533 data points, the average logarithmic RMS deviation is found to be about 12.1%.

3 H-mode temperature model

3.1 Model with width based on magnetic and flow shear stabilization

This model is given by Eq. (13) in Ref. [5]. It was assumed in this model that the pressure gradient within the pedestal region is constant and limited by the first stability of the ballooning mode. Then, the total pressure at the top of the pedestal ($p_{\rm ped}$) is

$$p_{\rm ped} \equiv 2 n_{\rm ped} k T_{\rm ped} = \Delta \left(\frac{\partial p}{\partial r} \right)_c \tag{4}$$

where $n_{\rm ped}$ and $T_{\rm ped}$ are the density and temperature at the top of the pedestal, k is the Boltzmann's constant, Δ is the pedestal width and $(\partial p/\partial r)_c$ is the critical pressure gradient of ballooning mode. Rewriting Eq. (4), one can obtain the value of $T_{\rm ped}$,

$$T_{\rm ped} = \frac{1}{2kn_{\rm ped}}\Delta\left(\frac{\partial p}{\partial r}\right)_c , \tag{5}$$

given the value of the pressure gradient and the width of the pedestal region.

In this model, the width of the pedestal, Δ, is assumed to be determined by a combination of magnetic and flow shear stabilization of drift modes [4],

$$\Delta = C_W \rho s^2. \tag{6}$$

where s is the magnetic shear, ρ is the ion gyro-radius at the inner edge of the steep gradient region of the pedestal and C_W is a constant of proportionality chosen to optimize the agreement with experimental data. The first stability ballooning mode limit is approximated by

$$\left(\frac{\partial p}{\partial r}\right)_c \equiv -\left(B_T^2/2\mu_0 Rq^2\right)\ \alpha_c \approx -\left(B_T^2/2\mu_0 Rq^2\right)\left(0.4s\left(1+\kappa_{95}^2\left(1+5\delta_{95}^2\right)\right)\right), \tag{7}$$

where the magnetic q and shear s are evaluated one pedestal width away from the separatrix, R is the major radius, B_T is the vacuum toroidal magnetic field evaluated at major radius R, κ_{95} is the elongation at the 95% magnetic surface ($\kappa_{95} = 0.914\kappa_x$, where κ_x is the elongation at the separatrix) and δ_{95} is the triangulrity at the 95% magnetic surface ($\delta_{95} = 0.914\delta_x$, where δ_x is the triangularity at the separatrix).

After combining Eqs. 5, 6 and 7 with some algebra, the following expression can be obtained for the pedestal temperature [5]:

$$T_{\rm ped} = C_W^2\left(\left(\frac{4.57\times10^{-3}}{4\mu_0(1.6022\times10^{-16})}\right)^2\left(\frac{B_T^2}{q^4}\right)\left(\frac{A_H}{R^2}\right)\left(\frac{\alpha_c}{n_{\rm ped}}\right)^2 s^4\right) \tag{8}$$

where $T_{\rm ped}$ is the pedestal temperature in the unit of keV, A_H is the average hydrogenic ion mass in the unit of AMU and $n_{\rm ped}$ is the electron density at the top of the pedestal in units of m^3.

The magnetic q has a logarithmic singularity at the separatrix. At one pedestal width away from the separatrix, the magnetic q is approximated by

$$\begin{aligned} q(r) \quad = \quad & \left(\tfrac{0.85a^2B_T}{IR}\right)\left(\tfrac{1+\kappa_{95}^2(1+2\delta_{95}^2-1.2\delta_{95}^3)(1.17-0.65a/R)}{[1-(a/R)^2]^2}\right) \\ & \left\{\left[1+\left(\tfrac{r}{1.4R}\right)^2\right]^2+0.27\left|\ln\left(\tfrac{1-r}{a}\right)\right|\right\}, \end{aligned} \tag{9}$$

where $r = a - \Delta$ is the position of the top of the pedestal and I is the plasma current. The magnetic shear, $s = \partial \ln q/\partial \ln r$, which is computed using the magnetic q from Eq. (9), is then reduced by the effect of the bootstrap current, as described in Ref. [5]. Since the pedestal width is needed to compute the magnetic q, the magnetic shear, s, and the normalized pressure gradient α_c, and since the pedestal width is a function of the pedestal temperature, the right hand side of Eq. (8) for the pedestal temperature depends nonlinearly on the pedestal temperature. Consequently, a non-linear equation solver is required to solve Eq. (8) to determine $T_{\rm ped}$.

The coefficient C_W in the expressions for the pedestal width [Eq. (6)] and the pedestal temperature [Eq. (8)] is determined by calibrating the model for the pedestal temperature against 533 data points for type I ELMy H-mode plasmas obtained from the International Pedestal Database version 3.1, using discharges from ASDEX-U, DIII-D, JET, and JT-60U tokamaks, as described in Ref. [5]. Ion temperature measurements were used for the pedestal temperature whenever they were available. However, this pedestal temperature model does not distinguish between electron and ion temperature. It is found that the value $C_W = 2.42$ yields a minimum logarithmic RMS deviation of about 32.0% for this data.

3.2 Model with width based on flow shear stabilization

This pedestal temperature model, which is given by Eq. (19) in Ref. [5], employs similar approach from section 3.1, but with different scaling of the width of the pedestal. The width of the pedestal, Δ, is derived from the assumption that the $\mathbf{E}_{\mathrm{r}} \times \mathbf{B}$ suppression of long wavelength modes is assumed to be the relevant factor in establishing the edge transport barrier. The local growth of the long wavelength modes can be estimated by sound speed divided by the connection length between the bad curvature region, the destabilizing curvature region on the outer side of the torus, and the good curvature region, the stabilizing curvature region on the inner side of the torus, in the pedestal region. The following result for the pedestal width is obtained:

$$\Delta = C_W \sqrt{\rho R q}. \tag{10}$$

where ρ is the gyro-radius, R is the major radius, q is the safety factor and C_W is a constant of proportionality chosen to optimize the agreement with experimental data.

After combining Eqs. 5, 7 and 10 with some algebra, the following expression can be obtained for the pedestal temperature in the unit of keV:

$$T_{\mathrm{ped}} = C_W^{4/3} \left(\left(\frac{(4.57 \times 10^{-3})^{1/2}}{4\mu_0 (1.6022 \times 10^{-16})} \right)^{4/3} \left(\frac{B_T}{q} \right)^2 \left(\frac{\sqrt{A_H}}{R} \right)^{2/3} \left(\frac{\alpha_c}{n_{\mathrm{ped}}} \right)^{4/3} \right) \tag{11}$$

where A_H is the average hydrogenic ion mass in AMU and n_{ped} is the electron density at the top of the pedestal in units of m^3. Note that Eq. 11 is a non linear equation as explained in section 3.1. The coefficient C_W in the expressions for the pedestal width [Eq. (10)] and the pedestal temperature [Eq. (11)] is determined by calibrating the model for the pedestal temperature against 533 data points for type I ELMy H-mode plasmas obtained from the International Pedestal Database version 3.1, using discharges from ASDEX-U, DIII-D, JET, and JT-60U tokamaks, as described in Ref. [5]. It is found that the value $C_W = 0.22$ yields a minimum logarithmic RMS deviation of about 30.8% for this data [5].

3.3 Model with width based on normalized poloidal pressure

This pedestal temperature model, which is given by Eq. (29) in Ref. [5], employs similar approach with the model in section 3.1, but uses different scaling of the pedestal width. The width of the temperature pedestal, Δ, is taken from the width scaling that fits to DIII-D database [6],

$$\Delta = C_W \sqrt{\beta_\theta} R. \tag{12}$$

where β_θ is the poloidal normalized pressure, R is the major radius and C_W is a constant of proportionality chosen to optimize the agreement with experimental data. In the steep gradient region of the pedestal, the pressure gradient is assumed to be constant and to be limited by the ideal, short-wavelength, ideal MHD ballooning limit, which is described in Eq. 7 above.

After combining Eqs. 5, 7 and 12 with some algebra, the following expression can be obtained for the pedestal temperature in the unit of keV:

$$T_{\mathrm{ped}} = C_W^2 \left(\left(\frac{1}{4\mu_0 (1.6022 \times 10^{-16})} \right) \left(\frac{B_T}{q^2} \right)^2 \left(\frac{R}{a} \right)^2 \left(\frac{\alpha_c^2}{n_{\mathrm{ped}}} \right) \left(\frac{\pi \; q_{95} (1 + \kappa_{95})}{5 g_s} \right)^2 \right) \tag{13}$$

where n_{ped} is the electron density at the top of the pedestal in units of m^3, q_{95} is the safety factor at 95% flux surface and g_s is the shaping factor, which is defined as

$$g_s = \frac{(1 + \kappa_{95}^2 \left(1 + 2\delta_{95}^2 - 1.2\delta_{95}^3\right) (1.17 - 0.65 a/R)}{(1 - (a/R)^2)^2}. \tag{14}$$

Note that Eq. 13 is a non linear equation as explained in section 3.1. The coefficient C_W in the expressions for the pedestal width [Eq. (12)] and the pedestal temperature [Eq. (13)] is determined by calibrating the model for the pedestal temperature against 533 data points for type I ELMy H-mode plasmas obtained from the International Pedestal Database version 3.1, using discharges from ASDEX-U, DIII-D, JET, and JT-60U tokamaks, as described in Ref. [5]. It is found that the value $C_W = 0.021$ yields a minimum logarithmic RMS deviation of about 32.9% for this data [5].

3.4 Thermal Conduction Model I

This model is given by Eq. (2) in Ref. [8]. If the pedestal stored energy, $W_{\rm ped}$, is known, the pedestal temperature can be found as

$$T_{\rm ped} = \frac{W_{\rm ped}}{2kn_{\rm ped}(0.92V)} \tag{15}$$

where V is the plasma volume. Note that the constant of 0.92 is the fraction of the total volume occupied by the pedestal [7]. The plasma volume can be estimated as

$$V \approx 2\pi R a^2 \kappa \tag{16}$$

By fitting to all types of ELMy H-mode discharges in the pedestal database DB3V2 [8], it was found that the pedestal stored energy is

$$W_{\rm ped,1} = 0.00064 I^{1.58} R^{1.08} P_{\rm loss}^{0.42} n_{19}^{-0.08} B_T^{0.06} \kappa^{1.81} \epsilon^{-2.13} A_H^{0.20} F_{\rm q}^{2.09} \tag{17}$$

where I is the plasma current in unit of MA, $P_{\rm loss}$ is the loss power in unit of MW, n_{19} is the density in the unit of 10^{19} particles per m^3, ϵ is the inverse aspect ratio and $F_{\rm q}$ is the shaping factor ($\equiv q_{95}/q_{\rm cyl}$, where q_{95} is the safety factor at 95% flux surface and $q_{\rm cyl}$ is the cylindrical safety factor defined as $5\kappa a^2 B/RI$). This formula yields the RMSE of 23.5% with the data [8]. This scaling satisfies both the Kadomtsev and the gyro-Bohm constraints with $B\tau_{ped} \propto \rho *_{\rm ped}^{-3} \beta_{\rm ped}^{-1.3}$.

By combining Eqs. 15, 16 and 17, the pedestal temperature in unit of keV can be found as

$$T_{\rm ped} = (8.86 \times 10^{-21}) I^{1.58} R^{0.08} P_{\rm loss}^{0.42} n_{19}^{-0.08} n_{\rm ped}^{-1.0} B_T^{0.06} \kappa^{0.81} a^{-2} \epsilon^{-2.13} A_H^{0.20} F_{\rm q}^{2.09} \tag{18}$$

3.5 Thermal Conduction Model II

In this model, a similar approach in section 3.4 is used. By fitting to only type I ELMy H-mode discharges in the pedestal database DB3V2 [8], it is found that the pedestal stored energy is

$$W_{\rm ped,2} = 0.0081 I^{1.41} R^{1.37} P_{\rm loss}^{0.50} n_{19}^{-0.15} B_T^{0.32} \kappa^{1.21} A_H^{0.20} F_{\rm q}^{1.61} \kappa^{1.21} \tag{19}$$

This formula yields the RMSE of 18.1% with the data [8]. By combining Eqs. 15, 16 and 19, the pedestal temperature in unit of keV can be found as

$$T_{\rm ped} = (4.38 \times 10^{-20}) I^{1.41} R^{0.37} P_{\rm loss}^{0.50} n_{19}^{-0.15} n_{\rm ped}^{-1.00} B_T^{0.32} \kappa^{0.21} a^{-2} A_H^{0.20} F_{\rm q}^{2.09} \tag{20}$$

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